Section 2.2 - Derivative as a function
(Rate of change)

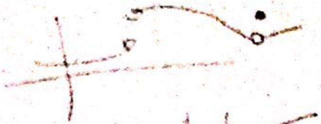
↳ derivative at any c - across the whole function

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- Where is there no derivative?

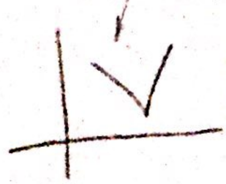
At a:

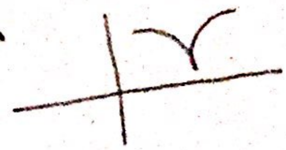
- discontinuity



- vertical Tangent

- corner / cusp





- Differentiability implies continuity
  ↳ but continuity ≠ differentiability

- GRAPHING

f		f'
tan is horizontal	→	x-intercept
+ slope	→	above x-axis
− slope	→	below x-axis

- find the rate of change for any c

- $f(x) = -4$ $\quad \lim_{h \to 0} \frac{-4 - 4}{h} = 0$

- $f(x) = 3x - 5$ $\quad \lim_{h \to 0} \frac{3(\cancel{x}+h) - \cancel{5} - \cancel{3x} + \cancel{5}}{h} = \lim_{h \to 0} \frac{3\cancel{h}}{\cancel{h}} = 3$

- $f(x) = 2x^2 + 4$ $\quad \lim_{h \to 0} \frac{2(x+h)^2 + \cancel{4} - 2x^2 - \cancel{4}}{h} = \lim_{h \to 0} \frac{2(\cancel{x^2} + 2xh + h^2) - \cancel{2x^2}}{h}$

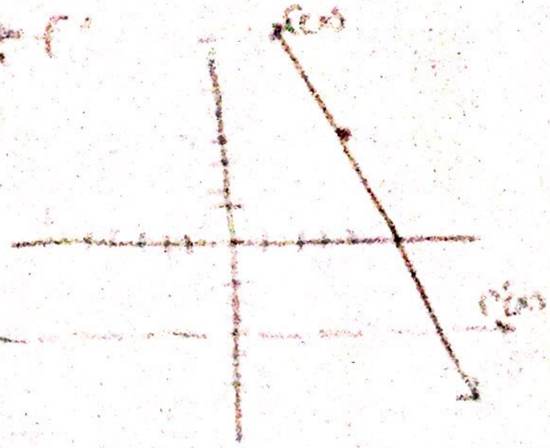
$\lim_{h \to 0} \frac{4xh + 4h^2}{h} = \lim_{h \to 0} 4x + 4h^2$

$= 4x$

- differentiate & graph both f & f'

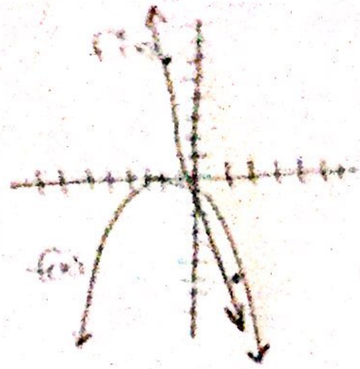
- $f(x) = -4x-5$

  $f'(x) = \lim\limits_{h\to 0} \dfrac{-4(x+h)-5+4x+5}{h} = -4$



- $f(x) = -3x^2+2$

  $f'(x) = \lim\limits_{h\to 0} \dfrac{-3(x+h)^2+2+3x^2-2}{h} = -6x$



- Does the function have a derivative at c? if it does, what is $f'(c)$
  if not, why?

- $2x^{1/3}$ $c=0$ NO, vertical asymptote
- $f(x) = |x^2-4|$ at $c=-2$ NO, cusp

- what is c & what is f?

- $\lim\limits_{h\to 0} \dfrac{(2+h)^3-8}{h}$ $c=2$ $f=x^3$

- $\lim\limits_{x\to 1} \dfrac{x^{11}-1}{x-1}$ $c=1$ $f=x^{11}$

- $\lim\limits_{x\to \frac{\pi}{4}} \dfrac{\cos(x)-\frac{\sqrt{2}}{2}}{x-\frac{\pi}{4}}$ $c=\pi/4$ $f=\cos x$

- $\lim\limits_{x\to 0} \dfrac{3x^3-2x}{x}$ $c=0$ $f=3x^3-2x$