Section 2.1 – Rates of change & the DERIV

$t_1 \neq t_0$

- Average Velocity (AV) – $\overset{\text{distance}}{\Delta s} \over \underset{\text{time}}{\Delta t}$ $= \frac{f(t_1) - f(t_0)}{t_1 - t_0}$ $(\Delta t = t_1 - t_0)$

- Instantanious Velocity (V) – $\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \lim_{t \to t_0} \frac{f(t) - f(t_0)}{t - t_0}$

- Equation of Tangent line to a function point

- Slope sec line eq? $\longrightarrow$ $m_{sec} = \frac{f(x) - f(c)}{x - c} = \frac{\Delta y}{\Delta x}$

- slope tan line $\rightarrow$ $\lim_{x \to c} \frac{f(x) - f(c)}{x - c} = f'(c)$ ← need a point

line eq: $y - f(c) = f'(c)(x - c)$

$y = mx + b$

$\rightarrow$ = instantanious rate of change = derivative

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Examples

- Find Eq. of Tan line:

(14) • $f(x) = x^2 + 2$ $\quad \overset{(x,y)}{(-1, 3)}$

$$\lim_{x \to -1} \frac{x^2 + 2 - (1+2)}{x + 1} = \lim_{x \to -1} \frac{\overset{(x+1)(x-1)}{x^2 - 1}}{x + 1}$$

$$\lim_{x \to -1} x - 1 = -2$$

$3 = -2(-1) + b$

$3 = -3 + b$

$6 = b$

$y = -2x + 6$

8) • $f(x)=\sqrt{x}$ $\overset{x,\ y}{(4,2)}$ $\lim\limits_{x\to 4} \frac{\sqrt{x}-\sqrt{4}}{x-4} = \lim\limits_{x\to 4} \frac{\sqrt{x}-\sqrt{4}}{x-4} \frac{(\sqrt{x}+\sqrt{4})}{\sqrt{x}+\sqrt{4}}$

$= \lim\limits_{x\to 4} \frac{\cancel{x-4}}{\cancel{x-4}(\sqrt{x}+\sqrt{4})} = \lim\limits_{x\to 4} \frac{1}{\sqrt{x}+\sqrt{4}} = 4$

$2=4(4)+b$

$2=16+b$

$-14=b$

$y=4x-14$

– what is the Rate of change @ the indicated #?

• $f(x)=x^2-1$ a) -1 b) $=1$ c) $=$ any real # (c)

a) $\lim\limits_{x\to -1} \frac{(x^2-1)-(-1^2-1)}{x--1} = \lim\limits_{x\to -1} \frac{x^2-1}{x+1}$ $\frac{\cancel{(x+1)}(x-1)}{\cancel{x+1}}$

$\lim\limits_{x\to -1}(x-1) = -2$

b) $\lim\limits_{x\to -1} \frac{(x^2-1)-(0)}{x-1} = \lim\limits_{x\to -1} x+1=0$

c) $\lim\limits_{x\to c} \frac{(x^2-1)-c^2+1}{x-c} = \lim\limits_{x\to c} \frac{x^2-c^2}{x-c}$ $\frac{(x+c)\cancel{(x-c)}}{\cancel{x-c}}$

$\lim\limits_{x\to c} x+c = 2c$

$(f'(1))$

– what is the derivative of the function at the number?

• $f(x)=3x-5$ @ 2 $\lim\limits_{x\to 2} \frac{3x-5-(6-5)}{x-2} = \lim\limits_{x\to 2} \frac{3x-6}{x-2}$

$\lim\limits_{x\to 2} \frac{3\cancel{(x-2)}}{\cancel{x-2}} = 3$

• $f(x)=2x^2+4$ @ 1 $\lim\limits_{x\to 1} \frac{2x^2+4-(2+4)}{x-1} = \lim\limits_{x\to 1} \frac{2x^2-2}{x-1}$

$\lim\limits_{x\to 1} \frac{2(x+1)\cancel{(x-1)}}{\cancel{x-1}} = 2(2) = 4$

• $f(x)=\frac{1}{x^2}$ @ 2 $\lim\limits_{x\to 2} \frac{\frac{1}{x^2}-\frac{1}{4}}{x-\frac{1}{4}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim\limits_{x\to 2} \frac{\frac{1}{x^3}-\frac{1}{4x}}{1-\frac{1}{4x}} = \frac{\frac{1}{2^3}-\frac{1}{4(2)}}{1-\frac{1}{4(2)}}$

$=0$