Section 4.5

L'Hôpital's Rule – if: you have limit $= \frac{0}{0}$ or $\frac{\infty}{\infty}$

Then: you can take the derivative of the top & bottom seperatly and still get the limit

Remember

$f \cdot g = \frac{f}{1/g}$

ex $\lim_{x \to 0} (x \ln x) = \lim_{x \to 0} \frac{\ln x}{1/x}$

Section 4.5 - Continued

L'Hôpitals - also works w/ one sided limits

Special Cases & what to do:

$0 \cdot \infty$:

$$f \cdot g = \frac{f}{1/g} \quad \text{or} \quad \frac{g}{1/f}$$

$\infty - \infty$: — also $-\infty + \infty$ or $\infty + (-\infty)$ Not $\infty + \infty$ or $-\infty + -\infty$

make a common denominator than apply L'Hôpitals

1^{∞}, 0^{0}, ∞^{0}

1) Take the natural log of both sides

$$\ln y = \ln [f(x)]^{g(x)} = g(x) \ln f(x)$$

can bring down exp.

2) take the limit (L'Hôpital)

3) if $\lim_{x \to c} \ln y = L \quad \lim_{x \to c} y = e^{L}$

e e

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28. $\lim_{x \to 1} \frac{2x^3+5x^2-4x-3}{x^3+x^2-10x+8} = \frac{0}{0} \rightarrow \lim_{x \to 1} \frac{6x^2+10x-4}{3x^2+2x-10} = \boxed{-\frac{12}{5}}$

36. $\lim_{x \to \infty} \frac{e^x}{x^4} = \frac{\infty}{\infty} \rightarrow \lim_{x \to \infty} \frac{e^x}{4x^3} \rightarrow \lim_{x \to \infty} \frac{e^x}{12x^2} \rightarrow \lim_{x \to \infty} \frac{e^x}{2\ldots}$

$\boxed{\lim_{x \to \infty} \frac{e^x}{24} = \infty}$

44. $\lim_{x \to \infty} (xe^{-x}) = 0 \cdot \infty \quad \lim_{x \to \infty} \frac{x}{e^x} = \frac{\infty}{\infty}$

$\boxed{\lim_{x \to \infty} \frac{1}{e^x} = 0}$

50. $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{e^x-1}\right) = \infty-\infty$

$$\lim_{x\to 0}\frac{e^x-1-x}{x(e^x-1)} = \frac{0}{0} \to \lim_{x\to 0}\frac{e^x-1}{xe^x+e^x-1} = \frac{0}{0} \to \lim_{x\to 0}\frac{e^x}{xe^x+e^x+e^x} = \frac{1}{2}$$

52. $\lim_{x\to 0} x^{x^2} = 0^0$

$$\lim_{x\to 0}$$

$$\ln y = \ln(x^{x^2}) = x^2 \ln x = 0\cdot\infty$$

$$\lim_{x\to 0}\frac{\ln x}{1/x^2} = \frac{\infty}{\infty} \to \lim_{x\to 0}\frac{1/x}{-2x^{-3}} = \lim_{x\to 0}\frac{x^2}{-2} = 0$$

$$\boxed{e^0 = 1}$$

56. $\lim_{x\to\infty} x^{1/x} = \infty^0$

$$\ln y = \ln x^{1/x} = \frac{1}{x}\ln x$$

$$\lim_{x\to\infty}\frac{1}{x}\ln(x) = \frac{\infty}{\infty} \to \lim_{x\to\infty}\frac{1/x}{1} = 0$$

$$\boxed{\lim_{x\to\infty} x^{1/x} = e^0 = 1}$$

58. $\lim_{x\to 0}(\cos x)^{1/x} = 1^\infty$

$$\ln(\cos x)^{1/x} = \frac{1}{x}\ln\cos x$$

$$\lim_{x\to 0}\frac{\ln(\cos x)}{x} = \frac{0}{0} \to \lim_{x\to 0}\frac{(1/\cos x)(-\sin x)}{1} = 0$$

$$\boxed{\lim_{x\to 0}(\cos x)^{1/x} = e^0 = 1}$$