Section 4.3 - Mean Value Theorem

Rolles Theorem - Proves mean value theorem

if: - f is continuous on $[a, b]$

- f is differentiable on (a, b)
- $f(a) = f(b)$

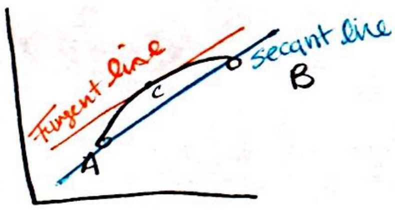
Then: there is at least one number, c, on the interval (a, b) where $f'(c) = 0$

Mean Value Theorem

if: - f is continuous on $[a, b]$

- f is differentiable on (a, b)

Then: There is at least one number, c, on the intervale where $f'(c) = \underbrace{\frac{f(b) - f(a)}{b - a}}_{m_{sec}}$



Corollarys:

- if $f'(x) = 0$ for all x on an interval than f (the original function) is a constant
- if $f'(x) = g'(x)$ than $f - g = $ constant
  i.e $f(x) = g(x) + c$

Increasing Decreasing function Test

$f'(x) > 0 \rightarrow$ increasing $\rightarrow +$

$f'(x) < 0 \rightarrow$ decreasing $\rightarrow -$

– Verify all 3 of Rolle's Theorem criteria are met than find c

Rule

- x^2+2x $[-2,0]$

1 ☑ continuous

2 ☑ differentiable on $[-2,0]$ $\to f'(x) = 2x+2$

3 ☑ $f(a) = f(b)$?

$-2^2+2(-2)=0$

$0+2(0)=0$

$c = 2x+2=0$

$c=-1$

- x^2+1 $[-1,1]$

1 ☑

2 ☑ $f'(x) = 2x$

3 ☑

$c=0$

$(-1)^2+1=2$

$(1)^2+1=2$

– Verify that the functions satisfy the criteria for MVT; find c

- $x+2+\frac{3}{x-1}$ $[2,7]$

☑ cont.

☑ diff

$f(a) = 2+2+\frac{3}{1} = 7$

$f(b) = 7+2+\frac{3}{6} = 9\frac{1}{2}$

$f'(x) = 1 + \frac{-3}{(x-1)^2}$

$1-\frac{3}{(x-1)^2} = \frac{9\frac{1}{2}-7}{7-2}$ $\frac{2.5}{5}$ $-\frac{3}{(x-1)^2} = \frac{1}{2}-1 = -\frac{1}{2}$

$\frac{(x-1)^2}{3} = 2$ $(x-1)^2 = 6$ $x-1 = \pm\sqrt{6}$

$x = \pm\sqrt{6}+1$

not in int.

$c = \sqrt{6}+1$

- xe^x $[0,1]$

☑ cont

☑ diff

$f(a)=0$

$f(1)=e$

$f'(x) = e^x + xe^x = e^x(1+x)$

$e^x(x+1) = e$

– where is the function increasing and decreasing

- $-x^3+3x^2+4$

$f'(x) = -3x^2+6x$

$-3x^2+6x=0$

$\frac{-6x}{-3x} = x$ $x=2$

$x=0$

$-$ 0 $+$ 2 $-$

@ $x=1$ $-3+6=+3$

@ $x=3$ $-3(9)+6(3) = -$

@ $x=-1$ $+3-6 = -$

- $e^x\cos(x)$ $0 \le x < 2\pi$

$f' = e^x\cos(x) - e^x\sin x$

$0 = e^x(\cos(x)-\sin(x))$

$\cos(x) = \sin(x)$

$\frac{\pi}{4}, \frac{5\pi}{4}$

$+$ $\pi/4$ $-$ $5\pi/4$ $-$

$x=0$ $e^0\cos(0) - e^0\sin 0 = 1$

$x=\frac{\pi}{2}$ $0 - e^{\pi/2}\sin(\frac{\pi}{2}) = -$

$x=\pi$