

Section 4.2 - Max, mins, critical numbers

ns:

absolute max - highest y -value of f
absolute min - lowest y -value of f

local max - highest around a point (c)
local min - lowest around a point (c)

Maxima - plural of maximum
Minima - plural of minima

Extremum - a max or min
Extrema - plural of extremum

'Global' = Absolute
'Relative' = local

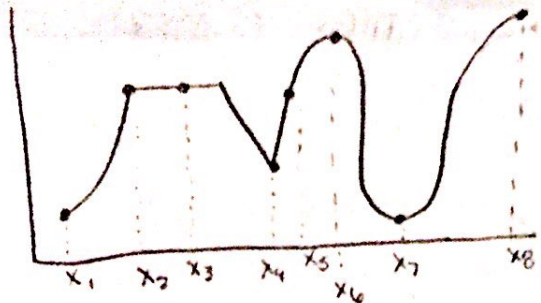
Critical Number
 $f'(c) = 0$ or $f'(c) = \text{undefined}$

Theorem's:

- Extreme Value Theorem (EVT)
 - if f is continuous on a closed interval $[a, b]$, then f obtains an absolute max $f(c)$ and min $f(d)$
- Fermat's Theorem
 - if f has a local extrema at c , and $f'(c)$ exists, then $f'(c) = 0$

Steps to solve this type of problem

- 1 - continuous?
- 2 - find critical values - in domain?
- 3 - evaluate original function at endpoints & critical values
- 4 - which is biggest? - max
- 5 - which is smallest? - min



- $x_1 = \text{LMin}$
- $x_2 = \text{LMX}$
- $x_3 = \text{N/LMX}$
- $x_4 = \text{LMin}$
- $x_5 = \text{N}$
- $x_6 = \text{LMX}$
- $x_7 = \text{AMin}$
- $x_8 = \text{AMax}$

— absolute min or max, local min or max, neither?

(14) Find critical #'s

• $f(x) = 1 - 6x + x^2$

$f'(x) = 2x - 6$
 $f'(0) @ x = 3$
 $f'(c) = \text{und - N/A}$

• $f(x) = x^3 - 6x$ $f'(x) = 3x^2 - 6$

$3x^2 - 6 = 0$
 $3x^2 = 6$
 $x^2 = 2$
 $x = \pm\sqrt{2}$

• $x^{1/3}$ $f'(x) = \frac{1}{3} x^{-2/3}$

$x = 0$

• $4 - \sqrt{x}$ $f'(x) = -\frac{1}{2} x^{-1/2} = -\frac{1}{2\sqrt{x}}$ $x = 0$

• $\frac{x}{x^2 - 1}$ $f'(x) = \frac{(x^2 - 1) - x(2x)}{(x^2 - 1)^2} = \frac{-x^2 - 1}{x^4 - 2x^2 + 1}$
 $(x^2 - 1)(x^2 - 1) = x^4 - 2x^2 + 1$ $x = 1$ $x = -1$

$f(x) = x^2 - 8x$ $[-1, 10]$ $f'(x) = 2x - 8$ $x = 4$

$x = -1 \rightarrow 9$

$x = 4 \rightarrow 16 - 32 = -16 \rightarrow \text{min}$

$x = 10 \rightarrow 10^2 - 8x = 20 \rightarrow \text{Max}$

$f(x) = 1 - 6x + x^2$ $[0, 4]$ $f'(x) = 2x - 6$ $x = 3$

$= 0 \rightarrow 1 \rightarrow \text{max}$
 $= 3 \rightarrow 1 - 6(3) + (3)^2 = 1 - 18 + 9 = -8$

$= 4 \rightarrow 1 - 6(4) + (4)^2 = 1 - 24 + 16 = -7 \rightarrow \text{min}$