3.2 - Implicit Differentiation & Inverse Trig

Implicit differentiation

1. take the derivative of both sides w/ respect to the independent var. — Remember Rules
2. use algebra to solve for $\frac{dy}{dx}$

- Slope of a tan line?
  $\hookrightarrow$ plug in x & y to eq to find slope

Rational Exponents

$\sqrt{x} \longrightarrow x^{1/2} \longrightarrow$ power rule!

$$\frac{d}{dx} x^{\frac{p}{q}} = \frac{p}{q} x^{(p/q)-1}$$

Inverse functions

$y = f(x)$ $\quad x = g(y)$

$$\frac{d}{dy} g(y_0) = \frac{1}{f'(x_0)}$$

- There is a point on the graph of f $\hookrightarrow (x_0, y_0)$
- There is a point on the graph of g $\hookrightarrow (y_0, x_0)$

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2} \qquad \frac{d}{dx} \cot^{-1} x = -\frac{1}{1+x^2}$$

$$\frac{d}{dx} \sec^{-1} x = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx} \csc^{-1} x = -\frac{1}{x\sqrt{x^2-1}}$$

- $y^4 - 4x^2 = 4$ $\quad \frac{dy}{dx} = \frac{2x}{4y^3}$

$4y^3 \frac{dy}{dx} - 8x = 0$

- $e^y = \tan x$ $\quad e^y \frac{dy}{dx} = \sec^2 x$ $\quad \frac{dy}{dx} = \frac{\sec^2 x}{e^y}$

- $e^y \cos x + e^{-x} \sin y = 10$

$\frac{dy}{dx} e^y \cos x + -\sin x e^y + -e^{-x} \sin y + \frac{dy}{dx} \cos y e^{-x} = 0$

$\frac{dy}{dx} = \frac{-\cos x(e^y) - (e)^{-x}\cos(y)}{\sin x e^y + e^{-x} \sin y}$

- $\frac{d}{dx} x^{1/3} - 1 = \frac{1}{3} x^{-2/3}$

- $\frac{d}{dx} \sqrt[4]{x^5} = \frac{d}{dx} (x^5)^{1/4} = \frac{d}{dx} x^{5/4} = \frac{5}{4} x^{1/4}$

- $\frac{d}{dx} \cos^{-1}(x^2) = \frac{d}{dx} \cos^{-1}(u) = -\frac{u'}{\sqrt{1-u^2}} = \frac{-2x}{\sqrt{1-x^4}}$

- $\frac{d}{dx} \sin^{-1}(1-x^2) = \frac{d}{dx} \sin^{-1}(u) = \frac{u'}{\sqrt{1-u^2}} = \frac{-2x}{\sqrt{1-(1-x^2)^2}}$

$u = 1-x^2$