

Section 3.1 - Chain Rule

- Chain Rule
din · dout (in)

- New Rules

- $\frac{d}{dx} a^x = a^x \ln a$ → be careful! Exponents other than x will result in a chain rule
- can take the power rule of a negative exp!

- Multiple Composite Functions

1. find inside function $\frac{1}{2}$ outside functions - inside called u (will have another function inside that - \checkmark)
2. rewrite original function w/ u subbed in (new f)
3. Take derivative of u - write out u & u'
↳ will have to use chain rule
4. Take derivative of new f
5. plug u & u' back in

Examples:

Take the derivative.

din ← in ↓
dout ↓
 $2 \cdot 3(2x+5)^2 = 6(2x+5)^2$

• $(2x+5)^3$

5. $3x^2(x^3-2)^4 = 15x^2(x^3-2)^4$

• $(x^3-2)^5$

2. $(e^x - x^2)^2 = 2(e^x - 2x)(e^x - x^2)$

• $(e^x - x^2)^2$

$$\sec^3(x) = (\sec(x))^3 \quad 3 \sec(x) \tan(x) (\sec(x))^2 = 3 \tan(x) \sec^3(x)$$

$$\cos(5x) = -5 \sin(5x)$$

$$e^{\frac{1}{x^2}} = \frac{-2x}{x^4} e^{\frac{1}{x^2}}$$

$$3^{\tan x} = \sec^2 x \cdot 3^{\tan x} \ln(3)$$

$$x^3 e^{2x} = 3x^2 e^{2x} + 2e^{2x} x^3$$

$$\frac{1}{1+2e^{-x}} = \frac{+2e^{-x}}{(1+2e^{-x})^2}$$

$$\frac{3}{x^5+2x^2-3} = \frac{-(5x^4+4x)}{(x^5+2x^2-3)^2}$$

$$x^{-5} = -5x^{-6}$$

$$\begin{aligned} \tan(e^{x^2}) & \quad \tan(u) & \quad u = e^{x^2} \\ & \quad \downarrow & \quad u' = 2xe^{x^2} \\ & \quad u' \cdot \sec^2(u) & \quad = 2xe^{x^2} \sec^2(e^{x^2}) \end{aligned}$$

$$\begin{aligned} e^{\csc^2(x)} & \quad e^u & \quad u = \csc(x)^2 \\ & \quad \downarrow & \quad u' = -2\csc(x)\cot(x) \\ & \quad u' e^u & \quad = -2\csc(x)\cot(x) e^{\csc^2(x)} \end{aligned}$$

$$\begin{aligned} 2\cos^2(x^2) & \rightarrow \cos(x^2)^2 & \quad u^2 & \quad u = \cos(x^2) \\ & \quad \downarrow & \quad u' = -2x\sin(x^2) \\ & \quad u' \cdot 2u & \quad = -4x\sin(x^2)(\cos(x^2)) \end{aligned}$$

$$\begin{aligned} e^{\pi x} \tan(\pi x) & \quad e^u \tan(u) & \quad u = \pi x \\ & \quad \downarrow & \quad u' = \pi \\ & \quad u' e^u \tan u + u' \sec^2(u) e^u & \quad = \pi e^{\pi x} \tan(\pi x) + \pi \sec^2(\pi x) e^{\pi x} \end{aligned}$$