

Section 2.4 - Products & Quotients; Higher order deriv.

Product Rule: $\frac{d}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$

Quotient Rule: $\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$ *Order matters*

Bonus Quotient Rule: $\frac{d}{dx} \left[\frac{1}{g(x)} \right] = \frac{-g'(x)}{[g(x)]^2}$

Higher Order Derivatives - derivative of the derivative

ex: $\frac{d}{dx} \frac{dy}{dx} = \frac{d^2y}{dx^2}$ - second order derivative

Rectilinear Motion: position $\rightarrow f(x)$
velocity $\rightarrow f'(x)$
acceleration $\rightarrow f''(x)$
 $\frac{ds}{dt} = \frac{dv}{dt}$ - over time

Galileo's Law: - position of an object in a vacuum; only exposed to gravity
 $s = -ct^2$

Examples:

• $f(x) = x^2 e^x$ $f'(x) = 2x e^x + e^x x^2$

• $f(x) = x^4 (x+5)$ $f'(x) = 4x^3(x+5) + x^4$

• $f(x) = (3x-2)(4x+5)$ $f'(x) = 3(4x+5) + 4(3x-2) = 24x+7$

$$\bullet F(u) = (u^4 - 3u^2 + 1)(u^2 - u + 2) \quad F'(u) = (4u^3 - 6u)(u^2 - u + 2) + (2u + 1)(u^4 - 3u^2 + 1)$$

$$\bullet f(x) = (x^2 + 1)(e^x + x) \quad f'(x) = (2x)(e^x + x) + (e^x + 1)(x^2 + 1)$$

$$\bullet f(z) = \frac{z+1}{2z} \quad f'(z) = \frac{2z - (z+1)(2)}{2z^2}$$

$$\bullet f(w) = \frac{1-w^2}{1+w^2} \quad f'(w) = \frac{(-2w)(1+w^2) - 2w(1-w^2)}{(1+w^2)^2}$$

$$\bullet G(u) = u^{-4} \quad G'(u) = \frac{-4u^3}{(u^4)^2}$$

$$\bullet f(x) = x^5 - \frac{5}{x^5} \quad f'(x) = 5x^4 - \frac{25x^4}{(x^5)^2}$$

$$\star \bullet f(x) = \frac{xe^x}{x^2 - x} \quad f'(x) = \frac{(e^x + xe^x)(x^2 - x) - (xe^x)(2x - 1)}{(x^2 - x)^2}$$

Find the first & 2nd Derivatives

$$\bullet f(x) = -5x^2 - 3x \quad f'(x) = -10x - 3 \quad f''(x) = -10$$

$$\bullet f(x) = x - e^x \quad f'(x) = 1 - e^x \quad f''(x) = -e^x$$

$$\bullet f(x) = 3x^4 e^x \quad f'(x) = (12x^3)e^x + (3x^4)e^x \quad f''(x) = (36x^2)e^x + 12x^3 e^x + 12x^3 e^x + 3x^4 e^x$$

$$\star \bullet f(x) = \frac{e^x}{x} \quad f'(x) = \frac{e^x x - e^x}{x^2} \quad f''(x) = \frac{(xe^x + e^x - e^x)x^2 - 2x(e^x x - e^x)}{(x^2)^2}$$

$$\bullet f(x) = x - \frac{1}{x} \quad f'(x) = 1 + \frac{1}{x^2} \quad f''(x) = \frac{-2x}{(x^2)^2}$$