

(4) (14 points) Find the instantaneous rate of change of $f(x)$ at $x = 2$. (Credit will not be awarded for use of derivative rules not yet covered in this course.)

$$f(x) = \sqrt{x-1} \quad @ \quad x=2$$

$$\lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} \rightarrow \lim_{x \rightarrow 2} \frac{\sqrt{x-1} - \sqrt{2-1}}{x-2} \rightarrow \lim_{x \rightarrow 2} \frac{\sqrt{x-1} - 1}{x-2}$$

$$\lim_{x \rightarrow 2} \frac{\sqrt{x-1} - 1}{x-2} \cdot \frac{(\sqrt{x-1} + 1)}{(\sqrt{x-1} + 1)} \quad \lim_{x \rightarrow 2} \frac{x-1-1}{x-2(\sqrt{x-1} + 1)} \quad \lim_{x \rightarrow 2} \frac{x-2}{x-2(\sqrt{x-1} + 1)}$$

$$\lim_{x \rightarrow 2} \frac{1}{\sqrt{x-1} + 1} = \frac{1}{\sqrt{2-1} + 1} = \frac{1}{1+1} = \frac{1}{2} \quad \checkmark$$

(5) (10 points) Use the results of problem (4) to find the equation of the line tangent to the graph of $f(x) = \sqrt{x-1}$ at the point $(2, 1)$.

#4 gave us the slope of the tangent line @ point $(2, 1)$

soo...

$$y - 1 = \frac{1}{2}(x - 2)$$

$$y - 1 = \frac{1}{2}x - 1$$

$$y = \frac{1}{2}x$$

(6) (16 points 8 each) A particle moves along the x -axis so that its position at time t is given by

$$x(t) = 2t^2 - 1.$$

(a) Find the average velocity of the particle over the time interval from $t = 0$ to $t = 2$.

$$\begin{aligned} \text{Average velocity} &= \frac{x(2) - x(0)}{2 - 0} \\ \text{from } t=0 \text{ to } t=2 &= \frac{(2(2)^2 - 1) - (2(0)^2 - 1)}{2 - 0} = \frac{(8 - 1) - (0 - 1)}{2} = \frac{7 + (-1)}{2} = \frac{8}{2} \\ &= 4 \checkmark \end{aligned}$$

(b) Find the instantaneous velocity of the particle at $t = 1$. (Note: No units are necessary.)

$$\text{Instantaneous velocity at } t=1 = \lim_{t \rightarrow 1} \frac{x(t) - x(1)}{t - 1}$$

$$\lim_{t \rightarrow 1} \frac{2t^2 - 1 - 1}{t - 1} \rightarrow \lim_{t \rightarrow 1} \frac{2t^2 - 2}{t - 1} \rightarrow \lim_{t \rightarrow 1} \frac{2(t^2 - 1)}{t - 1}$$

$$\lim_{t \rightarrow 1} \frac{2(t+1)\cancel{(t-1)}}{\cancel{t-1}} = \lim_{t \rightarrow 1} 2(t+1) = 2(1+1) = 2(2) = 4 \checkmark$$