Section 1.5 - infinite limits, limits at infinity, asymptotes

1

∞, $-\infty$ = not #'s

arithmatic

- $\infty + \infty = \infty$
- $\infty + c = \infty$
- $\infty \cdot c = \infty$
- $\infty \cdot -c = -\infty$
- $\frac{0}{\infty} = \frac{0}{-\infty} = 0$

<u>Not Defined</u>

$\infty - \infty$, $\frac{\infty}{\infty}$, $0 \cdot \infty$, ∞^0

2

$\lim_{x \to c} f(x) = \frac{K}{0}$ <u>may</u> be either $-\infty$ or ∞

- all + close to c: $\to \infty$
- all - close to c: $\to -\infty$
- can be either - limit DNE

3

if defined on (a, ∞) / $(-\infty, a)$

- $\lim_{x \to \infty} f(x) = L$
- $\lim_{x \to -\infty} f(x) = L$

[goal]

- $\lim_{x \to \pm\infty} \frac{K}{x^P} = 0$ Assuming P is defined $x < 0$
- $\lim_{x \to \infty} \ln(x) = \infty$
- $\lim_{x \to \infty} e^x = \infty$
- $\lim_{x \to 0} \ln(x) = -\infty$
- $\lim_{\theta \to \frac{\pi}{2}^-} \tan\theta = \infty$
- $\lim_{\theta \to \frac{\pi}{2}^+} \tan\theta = -\infty$

4

~Asymptotes~

- Vertical
  $x = c$ line
  - $\lim_{x \to c^\pm} f(x) = \pm\infty$
- Horizontal possibly (if denom = 0)
  $\lim_{x \to \pm\infty} f(x) = L$

2

$\lim_{x\to 1^-} \frac{2x+1}{x-1}$ $\lim_{x\to 1^-} 2x+1 = 2+1 = 3$

$= \frac{3}{0}$ $\lim_{x\to 1^-} x-1 = 0$

→ possibly infinite

because coming from - side $x<1 \to x-1<0$ & going through - numbers

$= -\infty$

3

- $\lim_{x\to\infty} \frac{5x^2+3x+1}{7x^2+5x+2} \frac{(1/x^2)}{(1/x^2)} = \lim_{x\to\infty} \frac{5+3/x+1/x^2}{7+5/x+2/x^2} = \frac{5}{7}$

find highest power in denominator & multiply/divide by reciprocal

- $\lim_{x\to\infty} \frac{\sqrt{7x^2+9x}}{3x+2} \frac{(1/x)}{(1/x)} = \frac{\frac{1}{\sqrt{x^2}}(\sqrt{7x^2+9x})}{3+2/x} = \frac{\sqrt{7+9/x}}{3+2/x} = \frac{\sqrt{7}}{3}$

$1/\sqrt{x^2}$ - for + #s

4

asymptotes of $\frac{2x^2-4x-6}{x^2-3x-4}$

H - take ∞ limit $\lim_{x\to\infty} \frac{2x^2-4x-6}{x^2-3x-4} \frac{(\frac{1}{x^2})}{(\frac{1}{x^2})} = 2$

V - has to approach some c - from 1 side - guess c

$x^2-3x-4 = \quad x=-1 \quad x=4$

$\lim_{x\to 1^+} \frac{2x^2-4x-6}{x^2-3x-4} = \frac{0}{0} \quad \frac{2(x-3)\cancel{(x+1)}}{x-4\cancel{(x+1)}} = \frac{8}{5}$

$\lim_{x\to 4^+} \frac{2x^2-4x-6}{x^2-3x-4} = \frac{10}{(4^+-4)(4^++1)^+}$ ← potential ±∞ $\quad \frac{10}{(+)(+)} = \infty$

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