inspiron

# Section 1.3 - Continuity

## Continuity at a point:

- $\lim_{x \to c} f(x) = f(c)$
  - f(c) is in the domain of f
  - $\lim_{x \to c} f_x$ exists

last section.

$\lim_{x \to c} P(x) = P(c)$ & $\lim_{x \to c} R(x) = R(c)$ - limit = function value
(c in domain)

Theorem: Every Rational function and polynomial is continuous at each number in its domain

## Removable Discontinuities

$\lim_{x \to c} f(x)$ exists but $\neq f(c)$

## Jump Discontinuities

$\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$

## One Sided Continuity - important for step functions

- continuous from the left: $\lim_{x \to c^-} f(x) = f(c)$
- from the right: $\lim_{x \to c^+} f(x) = f(c)$

[f is continuous, $f^{-1}$ is continuous]

[f & g are continuous  K = constant
- f+g  - f-g  - Kf  - fg  - $\frac{f}{g}$ $g(c) \neq 0$  are all continuous]

## Continuous on an interval

( - not included  [ - included

continuous on:

(a, b) - if continuous at each point in

(a, b] or [a, b) - if continuous on (a, b) & has one sided continuity at end point

## Compositions

$\lim_{x \to c} g(x) = L$ & f is continuous at g, than f is continuous at c

$\lim_{x \to c} (f \circ g)(x) = \lim_{x \to c} f(g(x)) = f(L)$

## IVT

- f is continuous on [a, b] and N is between f(a) & f(b), there is an x = c where f(c) = N

f(a) = + & f(b) = - there is an f(c) = 0 inbetween

## Is the function continuous at c? y = yes N = no

- $f(x) = x^3 - 5$ $c = 5$ (y)
- $f(x) = \frac{x}{x-2}$ $c = 2$ (N)
- $f(x) = \begin{cases} 2x+1 & x<0 \\ 2x & x>0 \end{cases}$ $c = 0$ (N)
- $f(x) = \begin{cases} 3x-1 & x<1 \\ 2 & x=1 \\ 2x & x>1 \end{cases}$ $c = 1$ (Y)
- $f(x) = \begin{cases} \vdots \\ 3x, & x>1 \end{cases}$ $c = 1$ (N)
- $f(x) = \begin{cases} x^2 & x<-1 \\ 2 & x=-1 \\ -3x+2 & x>-1 \end{cases}$ $c = -1$ (N)

## Continuous on the interval?

- $f(x) = 1 + \frac{1}{x}$ $[-1, 0)$ (Y)
- $f(x) = \sqrt{9-x^2}$ $[-3, 3]$ (Y)

## Where is the function continuous?

- $f(x) = x + 1 + \frac{2x}{x^2+5}$ $(-\infty, \infty)$
- $f(x) = \sqrt{x}\,(x^3 - 5)$ $[0, \infty)$
- $f(x) = \frac{x-4}{\sqrt{x}-2}$ $[0, 4) \cup (4, \infty)$
- $f(x) = \sqrt{\frac{4}{x^2-1}}$ $(x \mid x \neq -1, 1)$ – for all x
- $(x+2)^{1/2} = \sqrt{x+2}$ $[-2, \infty)$

## IVP – is there a zero on the interval? – or IVP gives no info

- $f(x) = x^4 - 1$ $[-2, 2]$ $f(-2) = -2^4 - 1 = +$ $f(2) = 2^4 - 1 = +$ – No info
- $f(x) = x^3 - 2x^2 - x + 2$ $[3, 4]$ $f(3) = +$ $f(4) = +$ no information
- $f(x) = \frac{x^2+3x+2}{x^2-1}$ $[-3, 0]$ – not continuous on interval – no info

## IVP – is there a zero?

- $f(x) = x^3 - 4x + 2$ $(1, 2)$ $f(1) = -$ $f(2) = +$ yes
- $f(x) = x^3 - x^2 - 2x + 1$ $(0, 1)$ $f(0) = +$ $f(1) = -$ yes
- $f(x) = x^3 - 6x - 12$ $(3, 4)$ $f(3) = -$ $f(4) = +$ yes
- $f(x) = x^4 - x^3 + x - 2$ $(1, 2)$ $f(1) = -$ $f(2) = +$ yes