

# Limits Continued - 1.2

## Properties of limits / limit laws

power theorem:

$$\lim_{x \rightarrow c} f(x)^n = L^n$$

$$\lim_{x \rightarrow c} x^n = c^n$$

root theorem

$$\lim_{x \rightarrow c} \sqrt[n]{f(x)} = \sqrt[n]{L}$$

allows you to take limits of

$$\lim_{x \rightarrow c} f(x) = L$$

$$\lim_{x \rightarrow c} g(x) = M$$

$M \neq 0$

polynomials

$$\lim_{x \rightarrow c} P(x) = P(c)$$

rational function

$$R(x) = \frac{P(x)}{Q(x)}$$

$$\lim_{x \rightarrow c} R(x) = R(c)$$

$\frac{1}{2}$   $c$  is in the domain  
(check the denominator)

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{L}{M}$$

when direct laws fail - indeterminate form  $\frac{0}{0}$   
- factor      - rationalize

$$\frac{\neq}{0} = \text{Undefined}$$

## Difference Quotient (limits of)

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad h \neq 0$$

1) plug in your function of  $x$       2) simplify      3) take the limit

$$\bullet \lim_{x \rightarrow -2} [3(x+1)] = 3(-2+1) = -3$$

$$\bullet \lim_{x \rightarrow -1} [x(x-1)(x+10)] = \lim_{x \rightarrow -1} x \cdot \lim_{x \rightarrow -1} (x-1) \cdot \lim_{x \rightarrow -1} (x+10) = -1(-2)(9) = 18$$

$$\bullet \lim_{x \rightarrow 0} (-3x+1)^2 = (-3(0)+1)^2 = 1$$

$$\bullet \lim_{x \rightarrow 8} \left( \frac{1}{4} \sqrt[3]{x} \right) = \frac{1}{4} \sqrt[3]{\lim_{x \rightarrow 8} x} = \frac{1}{4} \sqrt[3]{8} = \frac{1}{4} (2) = \frac{1}{2}$$

$$\bullet \lim_{t \rightarrow 2} \sqrt{3t+4} = \sqrt{\lim_{t \rightarrow 2} 3t+4} = \sqrt{6+4} = \sqrt{10}$$

$$\bullet \lim_{x \rightarrow 1} (2x^4 - 8x^3 + 4x - 5) = 2(1)^4 - 8(1)^3 + 4(1) - 5 = 2 - 8 + 4 - 5 = 6 - 13 = -7$$

$$\bullet \lim_{x \rightarrow 3} \frac{x^2+5}{\sqrt{3x}} = \frac{3^2+5}{\sqrt{9}} = \frac{14}{3}$$

$$\ast \bullet \lim_{x \rightarrow -2} \frac{x+2}{x^2-4} = \frac{0}{0} \quad \frac{x+2}{(x+2)(x-2)} = \frac{1}{x-2} = -\frac{1}{4}$$

$$\bullet \lim_{x \rightarrow -1} \frac{x^2+x^2}{x^2-1} = \frac{0}{0} \quad \frac{x^2(x+1)}{x+1(x-1)} = -\frac{1}{2}$$

$$\bullet \lim_{x \rightarrow 2} \left( \frac{3x}{x-2} - \frac{6}{x-2} \right) = \lim_{x \rightarrow 2} \left( \frac{3x-6}{x-2} \right) = \lim_{x \rightarrow 2} \frac{3(x-2)}{x-2} = 3$$

$$\bullet \lim_{x \rightarrow 3} \frac{\sqrt{x}-\sqrt{3}}{x-3} = \frac{0}{0} \quad \frac{(\sqrt{x}+\sqrt{3})}{\sqrt{x}+\sqrt{3}} = \frac{x-3}{x-3 \sqrt{x}+\sqrt{3}} = \frac{1}{\sqrt{x}+\sqrt{3}} = \frac{1}{2\sqrt{3}}$$

$$\bullet \lim_{x \rightarrow 3} \frac{\sqrt{x+1}-2}{x(x-3)} = \frac{0}{0} \quad \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} = \frac{x+1-4}{x(x-3)\sqrt{x+1}+2} = \frac{1}{x(\sqrt{x+1}+2)} = \frac{1}{12}$$

$$\bullet \lim_{h \rightarrow 0} \frac{3(x+h)+5 - (3x+5)}{h} = \lim_{h \rightarrow 0} \frac{3(x+h)+5 - (3x+5)}{h} = \frac{3x+3h+5 - 3x-5}{h} = \frac{3h}{h} = 3$$



# Limits - More Examples - Section 1.2

$$\bullet \lim_{x \rightarrow 2} (8 - 3x + 12x^2) = 8 - 3(2) + 12(2)^2 = 8 - 6 + 48 = 50$$

$$\bullet \lim_{t \rightarrow -3} \frac{6 + 4t}{t^2 + 1} = \frac{6 + 4(-3)}{(-3)^2 + 1} = \frac{6 - 12}{9 + 1} = \frac{-6}{10} = \frac{-3}{5}$$

$$\bullet \lim_{x \rightarrow 2} \frac{x^2 - 7x + 10}{x^2 - 4} = \frac{0}{0} = \frac{(x-2)(x-5)}{(x-2)(x+2)} \quad \lim_{x \rightarrow 2} \frac{x-5}{x+2} = \frac{-3}{4}$$

$$\bullet \lim_{x \rightarrow -1} \frac{x^2 + x - 2}{x^2 - 1} = \frac{0}{0} = \frac{(x+2)(x-1)}{(x+1)(x-1)} \quad \lim_{x \rightarrow -1} \frac{x+2}{x+1} = \frac{-1+2}{-1+1} = \text{DNE}$$

$$\bullet \lim_{x \rightarrow 5} \frac{x^2 + 2x - 35}{x^2 - 10x + 25} = \frac{0}{0} = \frac{(x+7)(x-5)}{(x-5)(x-5)} \quad \lim_{x \rightarrow 5} \frac{x+7}{x-5} = \frac{12}{0} = \text{DNE}$$

$$\bullet \lim_{x \rightarrow 25} \frac{5 - \sqrt{x}}{25 - x} \xrightarrow{\%} \frac{(5 + \sqrt{x})}{5 + \sqrt{x}} \cdot \frac{\lim_{x \rightarrow 25} (5 + \sqrt{x})}{\lim_{x \rightarrow 25} (25 - x)} = \frac{1}{10}$$

$$\bullet \lim_{x \rightarrow 9} \frac{9 - x}{\sqrt{x} - 3} \xrightarrow{\%} \frac{\sqrt{x} + 3}{\sqrt{x} + 3} \cdot \frac{\lim_{x \rightarrow 9} (\sqrt{x} + 3)}{\lim_{x \rightarrow 9} (9 - x)} = \frac{6}{0} = -\infty$$

$$\bullet \lim_{x \rightarrow 0} \frac{(x+3)^3 - 27}{x} \xrightarrow{\%} \frac{(x^3 + 9x^2 + 27x + 27) - 27}{x} = \frac{x^3 + 9x^2 + 27x}{x} \quad \lim_{x \rightarrow 0} \frac{x^2 + 9x + 27}{1} = 27$$

$$\bullet \lim_{x \rightarrow 0} \frac{x^2}{\sqrt{x^2 + 12} - \sqrt{12}} \xrightarrow{\%} \frac{x^2}{(\sqrt{x^2 + 12} + \sqrt{12})} = \frac{\lim_{x \rightarrow 0} x^2}{\lim_{x \rightarrow 0} (\sqrt{x^2 + 12} + \sqrt{12})} \quad \lim_{x \rightarrow 0} \sqrt{x^2 + 12} + \sqrt{12} = 2\sqrt{12}$$

$$\bullet \lim_{x \rightarrow 0} \frac{3}{x} \left( \frac{1}{5+x} - \frac{1}{5-x} \right) \stackrel{(c)}{=} \lim_{x \rightarrow 0} \frac{3}{x} \left( \frac{(5-x) - (5+x)}{(5+x)(5-x)} \right) = \lim_{x \rightarrow 0} \frac{3}{x} \left( \frac{-2x}{25 - x^2} \right) = \lim_{x \rightarrow 0} \frac{-6}{25 - x^2} = \frac{-6}{25}$$