Limits Continued - 1.2

Properties of limits / limit laws

power theorum:

$\lim_{x\to c} f(x)^n = L^n$ $\quad$ $\lim_{x\to c} x^n = c^n$ $\quad$ root theorum $\lim_{x\to c} \sqrt[n]{f(x)} = \sqrt[n]{L}$

→ allows you to take limits of polynomials $\lim_{x\to c} P(x) = P(c)$

$\lim_{x\to c} f(x) = L$ $\quad$ $\lim_{x\to c} g(x) = M$ $\quad$ $M \neq 0$

$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{L}{M}$

rational function

$R(x) = \frac{P(x)}{q(x)}$ $\quad$ $\lim_{x\to c} R(x) = R(c)$

; c is in the domain
(check the donominator)

when direct laws fail - indeterminate form $\frac{0}{0}$
- factor
- rationalize

$\frac{\#}{0}$ = undefined

Difference Quotient (limit of)

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \qquad h \neq 0$$

1) plug in your function of x 2) simplify 3) take the limit

- $\lim_{x\to -2} [3(x+1)] = 3(-2+1) = -3$
- $\lim_{x\to -1} [x(x-1)(x+10)] = \lim_{x\to -1} x \cdot \lim_{x\to 1} (x-1) \cdot \lim_{x\to 1} (x+10) = -1(-2)(9) = 18$
- $\lim_{x\to 0} (-3x+1)^2 = (-3(0)+1)^2 = 1$
- $\lim_{x\to 8} \left(\frac{1}{4}\sqrt[3]{x}\right) = \frac{1}{4}\sqrt[3]{\lim_{x\to 8} x} = \frac{1}{4}\sqrt[3]{8} = \frac{1}{4}(2) = 1/2$

- $\lim_{t\to 2}\sqrt{3t+4} = \sqrt{\lim_{t\to 2} 3t+4} = \sqrt{6+4} = \sqrt{10}$

- $\lim_{x\to 1}(2x^4-8x^3+4x-5) = 2(1)^4-8(1)^3+4(1)-5) = 2-8+4-5 = 6-13=-7$

- $\lim_{x\to 3}\frac{x^2+5}{\sqrt{3x}} = \frac{3^2+5}{\sqrt{9}} = \frac{14}{3}$

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- $\lim_{x\to -2}\frac{x+2}{x^2-4} = \frac{0}{0} \quad \frac{x+2}{(x+2)(x-2)} = \frac{1}{x-2} = -\frac{1}{4}$

- $\lim_{x\to -1}\frac{x^3+x^2}{x^2-1} = \frac{0}{0} \quad \frac{x^2(\cancel{x+1})}{\cancel{x+1}(x-1)} = -\frac{1}{2}$

- $\lim_{x\to 2}\left(\frac{3x}{x-2}-\frac{6}{x-2}\right) = \lim_{x\to 2}\left(\frac{3x-6}{x-2}\right) = \lim_{x\to 2}\frac{3(\cancel{x-2})}{\cancel{x-2}} = 3$

- $\lim_{x\to 3}\frac{\sqrt{x}-\sqrt{3}}{x-3} = \frac{0}{0} \quad \frac{(\sqrt{x}+\sqrt{3})}{\sqrt{x}+\sqrt{3}} = \frac{\cancel{x-3}}{\cancel{x-3}\sqrt{x}+\sqrt{3}} = \frac{1}{\sqrt{x}+\sqrt{3}} = \frac{1}{2\sqrt{3}}$

- $\lim_{x\to 3} = \frac{\sqrt{x+1}-2}{x(x-3)} = \frac{0}{0} \quad \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} = \frac{x+1-4}{x(x-3)\sqrt{x+1}+2} = \frac{1}{x(\sqrt{x+1}+2)} = \frac{1}{12}$

- $3x+5$ $\quad \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{3(x+h)+5-(3x+5)}{h} = \frac{\cancel{3x}+3h\cancel{+5}\cancel{-3x}\cancel{-5}}{h} = \frac{3h}{h} = 3$