limits - value that a function approaches as the input approaches some value

- use a secant line

$\frac{f(x)-f(c)}{x-c}$ $\frac{(y)}{x}$ and bring x closer & closer to c to make a tangent line — slope at that point

m_{tan} (slope) $= \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$ $\quad \lim_{x \to c} f(x) = L$

- left hand limit $\lim_{x \to c^-} f(x) = L_L$
- Right hand limit $\lim_{x \to c^+} f(x) = L_R$

3 limit Rules

1) the limit from the right and left must be equal for the limit to exist
2) the actual value at c and the limit of c can be different
3) It's possible for the limit to not exist

Properties of limits

- for any value of c...
  - if $f(x) = A$, $\lim_{x \to c} A = A$ (constant)
  - if $f(x) = x$, $\lim_{x \to c} x = c$
- If $\lim_{x \to c} f(x) = L$ $\quad \lim_{x \to c} g(x) = M$ $\quad K =$ constant
  - $\lim_{x \to c} (f(x) + g(x)) = L + M$
  - $\lim_{x \to c} Kf(x) = KL$
  - $\lim_{x \to c} (f(x) - g(x)) = L - M$
  - $\lim_{x \to c} (f(x)g(x)) = LM$

Examples:

- $\lim_{x \to 0} f(x)$ $\quad f(x) = \begin{cases} x, & x < 0 \\ 1 & x > 0 \end{cases}$ $\quad \lim_{x \to 0^-} x = 0$ $\quad \lim_{x \to 0^+} 1 = 1$ $\quad \boxed{\lim_{x \to 0} f(x) = DNE}$

- $\lim_{x \to 2} (2x+5)$ $\quad 2\lim_{x \to 2} x + \lim_{x \to 2} 5$ $\quad 2(2)+5 = 9$ $\quad \boxed{\lim_{x \to 2} (2x+5) = 9}$

- $\lim_{x \to 2} f(x)$ $f(x) = \begin{cases} 2x+5 & x \leq 2 \longrightarrow = 9 \text{ (Prev)} \\ 4x+1 & x > 2 \end{cases}$

$\rightarrow \lim_{x \to 2^+} (4x+1) = 4 \lim_{x \to 2} (x) + \lim_{x \to 2} (1) = 4(2) + 1$

$= 9$

$\boxed{\lim_{x \to 2} f(x) = 9}$